Ball-in-Tube Linearization Example

Lab 5: Nonlinear Control for a Flexible Joint

ECE 758: *Control System Implementation Laboratory*

Abstract

One of the laboratory design challenges is the balls-in-tubes experiment. In it, there are four tubes that each have a ball riding in them that is pushed up and down the tube by thrust generated by a fan. Here, we generate a simple model of a ball in a tube and show how feedback linearization allows for the application of linear control (e.g., PID control).

First, under a lift-coefficient hypothesis, assume that the thrust is proportional to the square of the voltage applied to the motor. That is,

$$T = Cv_{\text{in}}^2 \tag{1}$$

where T is the thrust generated by a van driven by voltage v_{in}. So long as the output impedance of the amplifier generated v_{in} is sufficiently low, we can assume that electrical resistance effects are negligible.

Next, use an *overly simple* point-mass model for the ball, as shown in Figure 1.

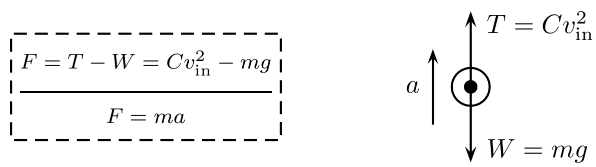

Figure 1: Simple point-mass model of a ball. Thrust T drives ball of mass m (weight W) with upward acceleration a.

In this model, the ball of mass m is driven upward by thrust T and pulled downward by gravity with weight $W = mg$. So the net upward force on the ball is $T - W$, which is equal to ma by Newton's second law, where a is the magnitude of the ball's upward acceleration. Hence, the ball's motion is modeled by

$$\overbrace{ma}^{F} = \overbrace{Cv_{\text{in}}^2}^{T} - \overbrace{mg}^{W}, \tag{2}$$

but $a = \dot{v} = \ddot{x}$, where x is the ball's relative position. Using position x as an output, Equation (2) is

$$\begin{cases} \dot{x} &= v \\ \dot{v} &= \frac{C}{m} v_{\text{in}}^2 - g. \end{cases} \tag{3}$$

For simplicity, force $v_{\text{in}} \geq 0$ and use $v_{\text{in}} = \sqrt{u}$ where $u \geq 0$. Hence, Equation (3) becomes

$$\begin{cases} \dot{x} &= v \\ \dot{v} &= -g + \frac{C}{m} u \end{cases} \qquad \left(\text{i.e., } \alpha(x,v) \triangleq -g \quad \text{and} \quad \beta(x,v) \triangleq \frac{C}{m}\right) \tag{4}$$

This system is already in *normal form.* Hence, without any coordinate transformation, it is immediately clear that this second-order system has relative degree 2 when position x is used as an output. So the control

$$u = \frac{m}{C}(w + g) \qquad \left(\text{i.e., } u = \frac{w - \alpha(x,v)}{\beta(x,v)}\right)$$

with $w \geq -g$ renders Equation (4) into the double-integrator LTI system

$$\begin{cases} \dot{x} &= v \\ \dot{v} &= w. \end{cases} \tag{5}$$

The parameter g is known ($9.8\,\text{m/s/s}$), the parameter m can be measured (e.g., with a scale), and the parameter C can be estimated from system data (e.g., by analyzing the acceleration of the ball when input u is constant). So the control

$$v_{\text{in}} = \sqrt{\frac{m}{C}(w + g)} \qquad \text{with} \qquad w \geq -g \tag{6}$$

linearizes the w–x system (and needs no *feedback* in this simple case). **Of course, the point-mass and lift-coefficient approximations may be overly naïve for this system.**