

Correction to Flexible Joint Model

Lab 4: Linear Quadratic Regulator (LQR) and Observer Design for Flexible Joint

ECE 758: Control System Implementation Laboratory

In the [Laboratory Design Challenges](#) document, the state-space model of the flexible joint has a *typo*, and so the state-space model will not produce the MATLAB results shown below it. **To remedy this problem, replace every R with R_m .** The effect of R is already included in the K_{stiff} parameter, and the typo omits any influence of the R_m parameter.

The correct model has

$$x = \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix}, \quad u = v_{\text{in}}, \quad \dot{x} = Ax + Bu, \quad \text{and} \quad y = Cx \quad (1)$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{K_{\text{stiff}}}{J_{\text{hub}}} & \frac{-K_m^2 K_g^2}{R_m J_{\text{hub}}} & 0 \\ 0 & \frac{-K_{\text{stiff}}(J_{\text{load}} + J_{\text{hub}})}{J_{\text{hub}} J_{\text{load}}} & \frac{K_m^2 K_g^2}{R_m J_{\text{hub}}} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{K_m K_g}{R_m J_{\text{hub}}} \\ \frac{-K_m K_g}{R_m J_{\text{hub}}} \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}. \quad (2)$$

These corrections have already been applied to the following MATLAB code.

```
A = [ 0, 0, 1, 0;
      0, 0, 0, 1;
      0, Kstiff/J_hub, -mpower(Km,2)*mpower(Kg,2)/(Rm*J_hub), 0;
      0, -Kstiff*(J_load+J_hub)/(J_hub*J_load), mpower(Km,2)*mpower(Kg,2)/(Rm*J_hub), 0 ];

B = [ 0; 0; Km*Kg/(Rm*J_hub); -Km*Kg/(Rm*J_hub) ];

C = [ 1 0 0 0; 0 1 0 0; 1 1 0 0 ];
```

This code uses `mpower(Km,2)` in place of `Km^2` so that you can copy and paste from this PDF into MATLAB. Using this code, your results should match the ones from the laboratory design challenge. That is,

```
format short e; A, B
```

should give you

```
A =
      0      0      1.0000e+00      0
      0      0      0      1.0000e+00
      0      7.6705e+02     -5.2795e+01      0
      0     -1.0401e+03      5.2795e+01      0

B =
      0
      0
      9.8333e+01
     -9.8333e+01
```