ECE 557: *Control, Signals, and Systems Laboratory*

Notes for Lab 4 (Gain Compensation and Feedback for a DC Servo)

1. Return time-domain system ID pre-lab and give some notes.

   - Notice differentiator.

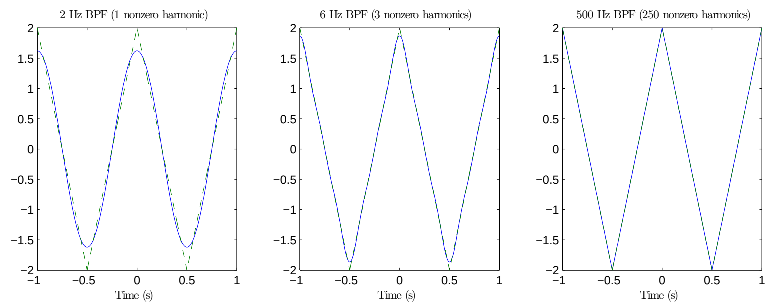
$$G_\omega(s) = \frac{\Omega(s)}{V_{\text{in}}(s)} = \frac{s\Theta(s)}{V_{\text{in}}(s)} = sG(s) = s\frac{1}{s(0.0026s + 0.1081)} = \frac{1}{0.0026s + 0.1081}$$

2. Return DSP lab reports and give some notes.

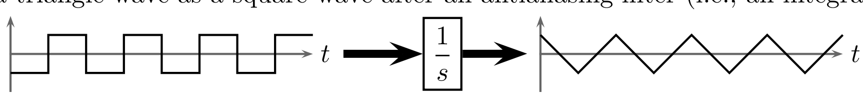
   - Quantization noise/error/distortion is not *directly* related to sampling and its effects.
     - An LSB for the lab is

$$\frac{20\,\text{V}}{2^{12}\,\text{codes}} \approx 4.88\,\text{mV/code} \quad \text{or} \quad \frac{20\,\text{V}}{2^{16}\,\text{codes}} \approx 0.31\,\text{mV/code}.$$

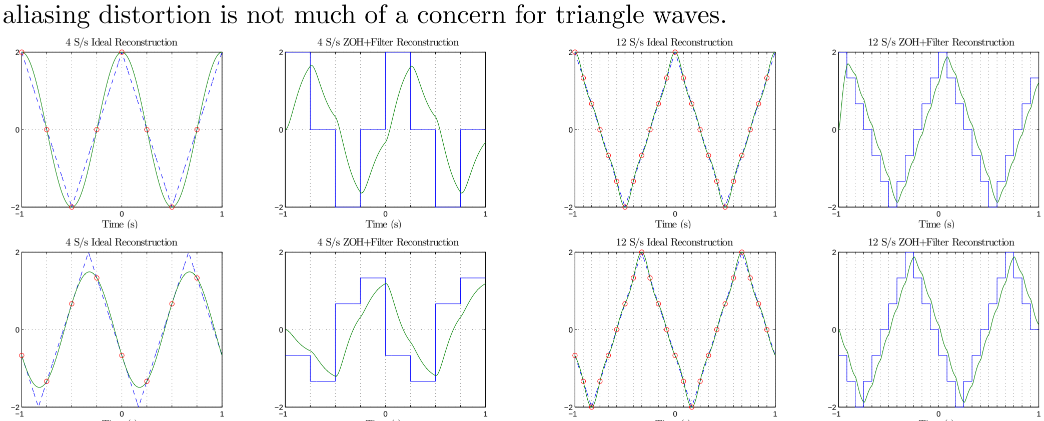
       So quantization effects are negligible for our experiments (i.e., other noise sources contribute more *and* we don't care about signals that small).
     - The term *resolution* is normally defined so that higher is better (e.g., "dots per inch", 1/LSB).
   - Triangle wave energy is tightly concentrated — error is less than 1.5% for a single harmonic.



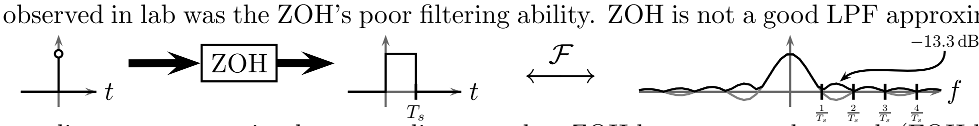
Think of a triangle wave as a square wave after an antialiasing filter (i.e., an integrator).



Troublesome harmonics are already attenuated. Further filtering would yield a pure sine wave. So aliasing distortion is not much of a concern for triangle waves.



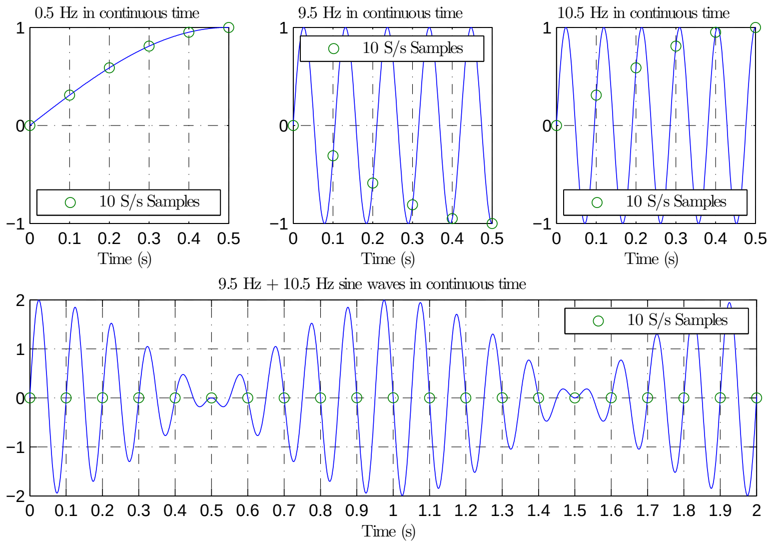
Issue observed in lab was the ZOH's poor filtering ability. ZOH is not a good LPF approximation.



Fast sampling puts separation between aliases, and so ZOH becomes good enough (FOH better).

- Sampling has *phase* effects from *negative* aliases. For example, at 10 S/s (i.e., folds over @ 5 Hz),

$$\begin{aligned} \sin(2\pi \times 9.5 \times t) &\quad \text{aliases to} \quad \sin(2\pi \times -0.5 \times t) = -\sin(2\pi \times 0.5 \times t), \\ \sin(2\pi \times 10.5 \times t) &\quad \text{aliases to} \quad \sin(2\pi \times 0.5 \times t), \\ \sin(2\pi \times 9.5 \times t) + \sin(2\pi \times 10.5 \times t) &\quad \text{aliases to} \quad -\sin(2\pi \times 0.5 \times t) + \sin(2\pi \times 0.5 \times t) \equiv 0. \end{aligned}$$



So aliasing *can* cause significant distortion!

- Pure *cosine* aliases *add* together because cos is an even function (i.e., all aliases are *positive*).
  - An *even* triangle wave at 1 Hz only has cosine harmonics at odd frequencies. When it is sampled at 4 S/s, *every* harmonic piles on top of 1 Hz and their amplitudes all *add* together.
  - A 4 S/s ideal reconstruction of such a triangle wave is a pure cosine with unity amplitude because aliasing *stretches* the available harmonics so that each sampled point is hit.

3. Page 14 of the lab text gives an example of working with *dSPACE ControlDesk* data in MATLAB.

```
load filename;                     % Loads file structure into filename
v = filename;                      % Copies filename structure to v
t = getfield(v.X(1), 'Data');      % Copies X time vector into t
y1 = getfield(v.Y(1), 'Data');     % Copies Y(1) vector into y1
y2 = getfield(v.Y(2), 'Data');     % Copies Y(2) vector into y2

mean( y1( find( t >= 4 ) ) )       % Average of Y(1) for time after 4 s

mean( y1( t >= 4 ) )               % Equivalent averaging statement
```

The book likes to do a lot of copying. There is no need to copy the whole file structure into `v` if you don't want to. Additionally, those `getfield` calls are not necessary because `Data` is a simple field name. So an alternative is:

```
load filename;                     % Loads file into  filename
t = filename.X(1).Data;            % Copies X time vector into t
y1 = filename.Y(1).Data;           % Copies Y(1) vector into y1
y2 = filename.Y(2).Data;           % Copies Y(2) vector into y2

mean( y1( t >= 4 ) )               % Average of Y(1) for time after 4 s
```

In fact, we can get rid of all copying and a few lines.

```
load filename;                                          % Loads up filename
mean( filename.Y(1).Data( filename.X(1).Data >= 4 ) )  % A mean shortcut
```

4. Feedback Control for Linear-Time-Invariant (LTI) Systems

- Open-loop control is not robust, and feedback controller can be simple because of loop dynamics.
- Consider unity-gain feedback with controller G_c, plant G_p, input signal X, and output signal Y.

$$Y = G_c G_p \overbrace{(X - Y)}^{\text{error}} = G_c G_p X - G_c G_p Y \implies \frac{Y}{X} = \frac{\overbrace{G_c G_p}^{\text{forward}}}{1 + \underbrace{G_c G_p}_{\text{loop}}} = \frac{G}{1+G} \quad \text{where} \quad G \triangleq G_c G_p$$

negative feedback

  - $G(s)$ is the *forward gain* (which is also the *loop gain* for *unity*-gain feedback).
    - Ideally, for Y to track X, G must be large at all frequencies and $G/(1+G)$ must be *stable*.
    - In reality, constraints (e.g., limited bandwidth or control energy) introduce *design trade-offs* (e.g., gain and bandwidth, speed and overshoot).
  - *System-type number* is how many *integrators* are in forward path. Improves *eventual* tracking.
    - For θ constant, $\dot{\theta} = 0$. If driving $\dot{\theta}$ with θ error, then a constant θ implies no θ error.
    - For no t_∞ error, controller's *imaginary-axis poles* must match signal's ("internal model").
      - With no error, controller gets *no input*, and so it must *naturally* behave like signal.
- Stability margins
  - Poles and zeros in G cause some frequencies to be delayed (i.e., phase shifted).
    - It's possible that one frequency ω_c will be delayed by 180° (i.e., multiplied by -1).
    - For that one frequency ω_c, negative feedback becomes *positive* feedback.
    - If $|G(\omega_c)| > 1$, positive feedback is constructive (i.e., closed-loop system is unstable).
    - Consider adjacent microphone and speaker — one tone *rings* and ***grows***.
      - To fix, sound engineer adjusts equalizer to attenuate speaker response at that frequency.
      - For frequency-dependent gain flexibility, we use *lead–lag* compensation (next week).
  - The *gain margin* is distance from 0 dB (i.e., where $|G| = 1$) at w_c (i.e., where $\angle G = 180°$).
    - It is how far we can increase gain before instability.
    - Must be *positive* for closed-loop stability.
  - The *phase margin* is distance from 180° where gain crosses 0 dB (i.e., where $|G| = 1$).
    - It is how much extra "delay" (phase) that can be added before instability.
    - Must be *positive* for closed-loop stability.
  - Wide margins ensure closed-loop stability will be *robust*.
    - Margins also represent distance between closed-loop poles and imaginary axis.
    - So margins have observable consequences.
  - Open-loop systems with 2 poles or less never cross 180° of phase shift/delay.
    - These systems are always stable — *gain margin* is ∞ and *phase margin* is always positive.
- Root locus: closed-loop pole position for every possible forward gain
  - Define K, $N(s)$, and $D(s)$ so that $G(s) = KN(s)/D(s)$. So open-loop (OL) zeros come from $N(s)$ and OL poles come from $D(s)$. Gain K has no impact on OL dynamics.
  - The closed-loop (CL) transfer function

$$\frac{G}{1+G} = \frac{K\frac{N(s)}{D(s)}}{1 + K\frac{N(s)}{D(s)}} = \frac{KN(s)}{D(s) + KN(s)} \xrightarrow{K \to \infty} 1$$

    - OL zeros and CL zeros are the same for all K.
    - For $K = 0$, CL poles match OL poles. As $K \to \infty$, CL poles move to OL (or CL) zeros.
      - $1/|s + a| \xrightarrow{|s| \to \infty} 0$, and so "zeros @ ∞" anchor "tent" for unmatched OL poles.
  - As long as CL poles are on left-hand side of complex plane, CL system is stable.
  - Use "tent" analogy with *stable* sliding CL poles to get snapshot of CL Bode magnitude.
    - As K grows, system *can react* faster ("bandwidth" increases). ¿Settling time/damping?

5. Complete the *Gain Compensation and Feedback for a DC Servo* lab
   - Implement gain compensation for position regulation of DC servo.
     - In *Simulink*, the Summing Junction (or *sum*) component is in the Math section of the library.
       - After placing it on your model, double-click on it.
       - Change the |++ to |+- to make one of the inputs negative.
         - The | is a placeholder for no input. It changes the *spacing* of the inputs.
         - The + means the input is positive, and the - means the input is negative.
   - Initially, use your pre-lab controller design.
     - *If you wish,* wire up a simulated system for comparison. Capture its output as well.
       - You might relate this to using an *estimator* (a subject of ECE 650 and ECE 750).
   - Using *dSPACE ControlDesk*, tune gain for fastest speed (i.e., highest gain) with $< 5\,\%$ overshoot.
     - Remember to press Enter (or ↩) to commit numeric input changes.
     - Set plotter's Y-axis for a Fixed scale with 0.95 minimum and 1.06 maximum to zoom in on important region of response.
     - Ideally, this type-1 system (i.e., system with one integrator) should have no steady-state error.
       - Unfortunately, nonlinearities and time variance in the system couple with quantization noise in the DAC and make the system far less than ideal. Unless gain is very high, steady-state error will be nonzero (e.g., motor stalls if it doesn't overcome starting friction).
       - Between the end of one run and the start of the next (i.e., when the motor is *not* being driven), manually reset the motor to its "zero position." This step improves system consistency (i.e., it helps with nonlinearities and time variance (e.g., "spots" of friction)).
       - Make sure gain is high enough for *steady-state* error to be less than $2\,\%$.
       - If you are having difficulty meeting both the $2\,\%$ steady-state requirement and the $5\,\%$ overshoot requirement, decrease gain substantially (i.e., *critically* damp or even *overdamp* your system with a gain *lower* than your pre-lab prediction).
   - Save data and record working gain choice.
   - AT ANY TIME, IF YOUR MOTOR STARTS CLICKING BACK AND FORTH VERY QUICKLY, STOP THE EXPERIMENT AND DISCONNECT THE MOTOR IF NECESSARY!! The high-frequency switching can cause **permanent damage** to the motor. (consider source of oscillations)

- Tips:
  - Do **work** out of directory on **local** hard drive — use as MATLAB working directory.
  - In *Simulink*, the hotkey for building a model is Ctrl - B .
  - Start *dSPACE ControlDesk* before doing *Simulink* builds.
  - In MATLAB, change *Termination* settings for DAC block.
    - Under *Termination* tab, check box to set 0 V stop value.
  - In *dSPACE* add a simState control.
    (i) Wire simState to 2-option radio button.
    (ii) Label one option Run and give value 2.
    (iii) Label other option Stop and give value 0.
    (iv) Set Capture Settings to *automatically* restart and set *capture time* to simulation time.

    Restart simulation as needed by using simState control (i.e., no need to change modes).
    (i) To stop early, change simState to Stop.
    (ii) Before restarting, tell Capture Settings to Stop and then Start.
      - This step clears the plotter data on the next run.
      - It will wait for you to change simState back to Run.
    (iii) When you're ready to start (e.g., after changing gains), set simState to Run.