Choosing an Output for Maximum Power: Impedance Matching

Abstract

Audio amplifiers warn users that speaker loads should be matched to the output impedance of the speaker drivers. Here, we explore why that is the case.

Consider the Thévenin equivalent in Figure ZM-1.

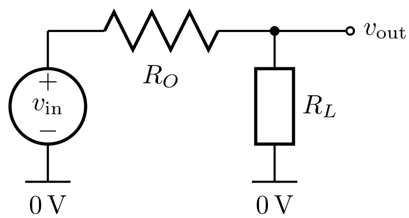

Figure ZM-1: Thévenin equivalent for voltage source (e.g., audio amplifier output).

The output voltage seen at the load is just a divided version of the input, and the power delivered to the load P_{out} is v_{out}^2/R_L. So

$$P_{\text{out}} = \frac{v_{\text{out}}^2}{R_L} = \frac{\left(\frac{R_L}{R_O+R_L} v_{\text{in}}\right)^2}{R_L} = \frac{R_L}{(R_O+R_L)^2} v_{\text{in}}^2. \tag{1}$$

For a fixed output resistance, a low load resistance will cause the divider to deliver very little potential to the load (and dissipate much power in the amplifier, possibly damaging it). Similarly, a high load resistance will deliver very little current to the load. Because power is the product of current and potential, there should be some intermediate load resistance that maximizes power delivered to the load.

Finding the Optimal Output Resistance

For a fixed load resistance R_L, let's find the optimal output resistance R_O^*. It's clear that Equation (1) is a monotonically decreasing function of R_O. That is, for a fixed R_L, P_{out} decreases as R_O increases. So the optimal output resistance is $0\,\Omega$ and the corresponding power delivered is v_{in}^2/R_L. This result is both unsurprising and unrealistic.

Finding the Optimal Load Resistance

Now, let's fix output resistance R_O and calculate the optimal load resistance R_L^*. We know that $R_L^* > 0$, and so the partial derivative of P_{out} with respect to R_L should be zero at $R_L = R_L^*$. In particular,

$$\left.\frac{\partial P_{\text{out}}}{\partial R_L}\right|_{R_L=R_L^*} = \left(\frac{-2R_L^*}{(R_O+R_L^*)^3} + \frac{1}{(R_L^*+R_O)^2} \right) v_{\text{in}}^2 = 0,$$

and so, assuming that $v_{\text{in}} > 0$,

$$\frac{-2R_L^*}{(R_O+R_L^*)^3} = \frac{1}{(R_L^*+R_O)^2}.$$

Thus,

$$2R_L^* = R_L^* + R_O.$$

So $R_L^* = R_O$. To verify, note that because $R_O > 0$,

$$\left.\frac{\partial^2 P_{\text{out}}}{\partial {R_L}^2}\right|_{R_L=R_L^*=R_O} = \frac{-2(R_L^*-2R_O)}{(R_O+R_L)^4} = -\frac{1}{8R_O^3} < 0,$$

and so $R_L^* = R_O$ is a maximum point (in fact, it's a global maximum). Note that the local "steepness" (i.e., convexity) of P_{out} around $R_L = R_L^* = R_O$ increases greatly as output impedance decreases, so impedance matching is less important for high impedance outputs (i.e., the high output impedance will dominate the low power delivery, like a current limiter[1]).

[1] Note that current limiters are typically implemented as current sources that are turned on above a certain limit. Not surprisingly, a current source has infinite input impedance.