ECE 209: *Circuits and Electronics Laboratory*

Notes for Lab 5 (Properties of Second-Order Circuits)

1. Systems review
   - $Me^{j\phi}$ can be represented as a radial vector of length M and angle ϕ from the origin.
     - The tip of the vector will be a point (x, y) and $Me^{j\phi} = x + jy$.
       * So vertical axis is imaginary (j) and horizontal axis is real.
       * By Pythagorean theorem (or distance formula), $M = \sqrt{x^2 + y^2}$.
       * By trigonometry, $\phi = \arctan2(y, x)$ (i.e., $\phi = \arctan(y/x)$ when (x, y) in quadrant I).
     - Also, $Me^{j\phi} = \cos(\phi) + j\sin(\phi)$.
       * Visualize a turntable with a single peg sitting at its edge.
       * As the turntable spins (i.e., as ϕ increases), the peg moves around a circle.
       * Imagine viewing the turntable from a side (rather than from the top).
         · From the side, peg position oscillates back and forth with a **sinusoidal** trajectory.
         · From two perpendicular sides, the two sinusoids are 90° out of phase — sine and cosine.
       * Alternatively, realize that $\cos(\phi)$ and $\sin(\phi)$ are the length of the adjacent and opposite sides of the triangle formed by $Me^{j\phi}$.
       * By the identity,

$$\cos(\phi) = \frac{e^{j\phi}+e^{-j\phi}}{2} = \mathrm{Re}\left(e^{j\phi}\right) \qquad \text{and} \qquad \sin(\phi) = \frac{e^{j\phi}-e^{-j\phi}}{2j} = \mathrm{Im}\left(e^{j\phi}\right)$$

         · Add a $\angle\phi$ vector to a $\angle{-\phi}$ vector — imaginary parts cancel and real parts double.
         · When thinking of $\sin(\omega t)$ and $\cos(\omega t)$, look at a ***rotating*** $e^{j\omega t}$ from the side.
       * Notice that $|e^{j\phi}| = \sqrt{(\mathrm{Re}(e^{j\phi}))^2 + (\mathrm{Im}(e^{j\phi}))^2} = \sqrt{\cos^2(\phi) + \sin^2(\phi)} = 1$.
     - This vector is called a **phasor** to differentiate it from vectors with complex elements.
       * Each element of such vectors can be represented as a phasor.
       * So we can have vectors of phasors (e.g., $\vec{v} = [a + jb, c + jd, M_1 e^{j\phi_1}, M_2 e^{j\phi_2}]$).
       * Phasors represented in polar (radial) form $M\angle\phi \triangleq Me^{j\phi}$ or Cartesian form $(\mathrm{Re}, \mathrm{Im})$.
   - Find transfer function from differential equation by assuming input $x(t) = Xe^{st}$ and output $y(t) = Ye^{st}$ where $s \triangleq \sigma + j\omega$ (i.e., steady-state solution).
     - Then $x' = sx$, $x'' = s^2x$, $y' = sy$, $y'' = s^2y$. For **these** functions, d/dt is multiplication by s.
     - Substitute derivatives and solve for $y(t)/x(t)$. Note that e^{st} cancels and $y(t)/x(t) = Y/X$, and your solution sets Y/X equal to a ratio of s polynomials.
     - For any s, complex value $H(s) \triangleq Y/X$ scales input to get output (i.e., $y(t) = H(s)Xe^{st}$).
     - $H(s)$ is the *transfer function.* Its denominator roots describe the transient response of system.
     - Transients must decay for system to reach steady-state. System is called **stable** in this case.
     - Because $e^{st} = e^{\sigma t}e^{\omega t}$, real part of each **pole** must be negative for stability.
       * "Poles" shape $|H(s)|$ by pulling it up to ∞ at points like a **tent pole.**
       * Numerator roots are called **zeros** because they pull tent to the ground.
     - To determine frequency response, put tent **poles** and **zeros** in complex plane and examine tent shape along imaginary axis (where Re = 0).
       * If poles have $\sigma = 0$, tent is pulled to infinity at that ω frequency (i.e., resonance).
       * Complex poles/zeros are in pairs. So tent is symmetric about real axis (i.e., Im = 0).
       * So for 2nd-order filter, **angle** of positive ("upper") pole determines *damping ratio* (ξ).
         · Note: $s^2 + 2\xi\omega_0 s + \omega_0^2$ — if $\xi < 1$, imaginary pole located at $\omega_0\angle(180° - \arccos(\xi))$.
         · *RLC* characteristic poly.: $s^2 + (R/L)s + 1/(LC)$ — LC sets ω_0; R sets damping ξ.

2. Filters review
   - **Corner/*cutoff*/"knee" frequency** is where **output *power*** is **half** of **passband power.**
     - If power gain is $1/2$, then signal gain is $1/\sqrt{2}$. Note that $20 \log_{10} \sqrt{2} = 10 \log_{10} 2 \approx 3\,\text{dB}$.
   - Pass- and stop-bandwidth are separated by "corners" at top of "transition regions."
   - For near ideal performance, usually want *steep* transitions with ***sharp*** *corners.*
   - Transition steepness governed by filter *order* (i.e., number of *poles*).
   - Corner sharpness governed by damping factors.
3. *Passive* second-order filters (voltage divider is electric bell with resistance damping)
   - To produce steep transitions, need high-order filters.
   - Low-order filters cannot be easily cascaded with active buffers.
   - Cascades of *RC* filters do not produce sharp knees ("many a soft knee do not a hard knee make").
   - Inductors and capacitors together (*LC* resonance) can greatly improve knee sharpness.
   - Inductors have many undesirable properties in small-signal (i.e., not **power**) circuits.
4. Second-order *active* filters.
   - *Active* filters allow for gain, simplicity, robustness, and can have hard "knees" without *inductors.*
   - Can construct Sallen-Key filter topology from modification of *RC* cascade.
     1. Start with *RC* section followed by second *RC* section (with *no buffer*).
        - To prevent second section from *loading* the first, buffer is usually inserted between them.
        - Buffered combination still has soft knee.
     2. Buffer (e.g., use unity-gain op. amp.) the output.
     3. Connect ground of first *RC* section to buffered output.
     4. This *bootstrapping* sharpens knee by applying second capacitor *gently* rather than all at once.
5. Introduce and complete the *Properties of Second-Order Circuits* lab.
   - Inductors are large tunable boxes in cabinet. Tune for 0.5 H. May measure R_L with DMM.
   - Generate step responses using **SLOW** square wave (e.g., ∼50 Hz).
     - Set scope to trigger on input channel. Adjust horizontal/vertical scales/position to zoom.
     - Capture **time scale** and **amplitude** information from scope.
   - If 741-type operational amplifier is not available, use 747 (two 741-type OAs on one chip).
     - 747 (and 741) part pinout on supplementary document.
     - **Note the supply rails!** Op. amps need **power** from **both sides** to be able to work.
     - Be sure to use **dual ±5 V supplies** with a **common ground**. Use ±6 V if you see clipping.
     - If you see a **lot of noise** on **OA output**, then add 1 μF (code `105`) across each supply rail.
       * Connect one **bypass** capacitor from **positive supply** to **ground.**
       * Connect one bypass capacitor from **negative supply** to **ground.**
       * Place capacitors **close** to operational amplifier pins and try to use short capacitor leads.
   - Resistor color codes: Black, Brown, ROYGBV, Gray, White correspond to **digits** 0–9
     - $\overset{5}{\text{Green}}$-$\overset{6}{\text{Blue}}$-$\overset{1}{\text{Brown}}$ = 560 Ω; $\overset{1}{\text{Brown}}$-$\overset{0}{\text{Black}}$-$\overset{3}{\text{Orange}}$ = 10 kΩ; $\overset{1}{\text{Brown}}$-$\overset{8}{\text{Gray}}$-$\overset{3}{\text{Orange}}$ = 18 kΩ
   - Capacitor codes are like resistor codes (digit_1/digit_2/number-of-zeros) but *typically* use unit pF
     - 0.01 μF = `103`; 0.1 μF = `104`; 0.068 μF = `683` or `0.068` (decimals usually indicate μF units)
   - **In your report,** for **both filters,**
     (i) Compare measured **step responses** to predicted curves from MATLAB.
     (ii) Plot the **frequency response** for the three cases in MATLAB.
       - Relate the frequency content of each filter to the corresponding step response.
       - For example, consider how steep the filter edge is (bandwidth) and how "ringy" the response is (damping and corner shape).

Sample Code

MATLAB Code for Step and Frequency Response of a *Single* Transfer Function

```
% Define transfer function parameters
R = 1e3;        % 1 kiloOhms (resistance)
L = 0.5;        % 0.5 Henry (inductance)
C = 0.01e-6;    % 0.1 microFarads (capacitance)

% Define Laplace-domain variable s (as transfer function object)
s = tf('s');

% Define new transfer function object using existing s object
H = ( 1/(L*C) )/( s^2 + (R/L)*s + 1/(L*C) );

% % Alternatively, could use vectors of numerator and denominator polynomial
% % coefficients:
% H = tf( [ 1/(L*C) ], [ 1 (R/L) 1/(L*C) ] );

% Generate step response in first figure
figure(1);
step( H );
grid on;

% Generate Bode plot in second figure
figure(2);
bode( H );
grid on;
```

MATLAB Code to Overlay *Several* Step and Frequency Responses

```
% Define three cases
R1 = 1e3; L1 = 0.5; C1 = 0.01e-6;
R2 = 10e3; L2 = 0.5; C2 = 0.02e-6;
R3 = 20e3; L3 = 0.5; C3 = 0.03e-6;

% Define Laplace-domain variable s (as transfer function object)
s = tf('s');

% Define three transfer functions
H1 = ( 1/(L1*C1) )/( s^2 + (R1/L1)*s + 1/(L1*C1) );
H2 = ( 1/(L2*C2) )/( s^2 + (R2/L2)*s + 1/(L2*C2) );
H3 = ( 1/(L3*C3) )/( s^2 + (R3/L3)*s + 1/(L3*C3) );

% Overlay three step responses in first figure
figure(1);
step( H1, 'b-', H2, 'g--', H3, 'm-.' );
grid on;
legend( 'H_1', 'H_2', 'H_3' );

% Overlay three Bode plots in second figure
figure(2);
bode( H1, 'b-', H2, 'g--', H3, 'm-.' );
grid on;
legend( 'H_1', 'H_2', 'H_3' );
```