ECE 209: *Circuits and Electronics Laboratory*

Notes for Lab 4 (Frequency Response of First-Order Active Circuits)

1. Comments on returned lab report.
   - Put units on tables and figures and show individual **data points** (i.e., not just interpolation).
   - In meters lab, **positive error** on R_v is a **good thing.** Manufacturer gives worst-case specification.

2. First-order *active* filters.
   - *Active* filters allow for gain, simplicity, robustness, and can have hard "knees" without inductors.
   - Use standard inverting/non-inverting OA configuration, but use (equivalent) Laplace-domain impedances instead of simple resistances.
   - To quickly determine characteristic of filter (i.e., low-pass, high-pass, or bandpass) consider what happens to OA configuration's "gain" at some sample frequencies.
     - In today's lab, both filter's have transfer function given by $-Z_F(s)/Z_I(s)$.
     - In the low-pass filter, $Z_I(s) = R_1$ and $Z_F(s) = R_2 \| (sC)^{-1}$.
       * $Z_F(0) \approx R_2$ for low frequencies — LF gain is $-R_2/R_1$ (i.e., R_2/R_1 with $-180°$ shift).
       * $Z_I(j\omega) \approx 0$ for high frequencies — HF gain is $-0/R_1 \approx 0$ (with $-180° - 90°$ shift).
       * First-order filter's time constant τ must depend on C, but because V_- is a virtual ground, the output does not "feel" the effect of R_1. So time constant $\tau = R_2 C$.
       * It is a low-pass filter with **passband gain** $K = -R_2/R_1$ and **time constant** $\tau = R_2 C$:

$$H_{\text{LPF}}(s) \triangleq \frac{K}{\tau s + 1} = \frac{-\frac{R_2}{R_1}}{sR_2C + 1}.$$

     - In the high-pass filter, $Z_I(s) = R_1 + 1/(sC)$ and $Z_F(s) = R_2$.
       * $Z_I(0) \approx \infty$ (i.e., "open" for VLF) — LF gain is $-R_2/\infty \approx 0$ (with $-180° + 90°$ shift).
       * $Z_I(j\omega) \approx R_1$ for high frequencies — HF gain is $-R_2/R_1$ (i.e., R_2/R_1 with $-180°$ shift).
       * First-order filter's time constant τ must depend on C, but because V_- is a virtual ground, the input does not "feel" the effect of R_2. So time constant $\tau = R_1 C$.
       * It is a high-pass filter with **passband gain** $K = -R_2/R_1$ and **time constant** $\tau = R_1 C$:

$$H_{\text{HPF}}(s) \triangleq \frac{Ks}{s + \frac{1}{\tau}} = \frac{K\tau s}{\tau s + 1} = \frac{-sR_2C}{sR_1C + 1}.$$

   - As in passive filters, **corner frequency** is where **output *power*** is **half** of **passband power.**
     - If power gain is 1/2, then signal gain is $1/\sqrt{2}$. Note that $20 \log_{10} \sqrt{2} = 10 \log_{10} 2 \approx 3\,\text{dB}$.

3. Introduce and complete the *Frequency Response of First-Order Active Circuits* lab.
   - If 741-type operational amplifier is not available, use 747 (two 741-type OAs on one chip).
     - 747 (and 741) part pinout on supplementary document.
     - **Note the supply rails!** Op. amps need **power** from **both sides** to be able to work.
     - Be sure to use **dual ± 5 V supplies** with a **common ground**. Use ± 6 V if you see clipping.
     - If you see a **lot of noise** on **OA output**, then add $1\,\mu$F (code `105`) across each supply rail.
       * Connect one capacitor from **positive supply** to **ground.**
       * Connect one capacitor from **negative supply** to **ground.**
       * Place capacitors **close** to operational amplifier pins and try to use short capacitor leads.
       * When used in this configuration, these capacitors are called **bypass capacitors** because they provide an alternate path for high-frequency (i.e., noisy) current. The operational amplifier sees steady rails because the ripples are directed elsewhere.

- The 741 is a **SLOW** OA. Its **bandwidth** and **slew rate** may corrupt high-pass filter data.
  - The *bandwidth* is typically 1.5 MHz, but it can be as low as 437 kHz. So signals approaching this frequency may see **attenuation** from poles within the OA that were previously negligible become significant. So **the high-pass filter may look like bandpass filter.**
  - The *slew rate* (i.e., maximum "speed" of OA output) is ∼0.5 V/μs (i.e., 500 kV/s).
    - "Speed" (i.e., derivative) of $A\sin(2\pi ft)$ output is $2\pi Af\cos(2\pi ft)$. Maximum is $2\pi Af$.
    - For the OA to be able to move quickly enough, $2\pi Af < 500\,\text{kV}/s$; so $f < (500\,\text{kV}/s)/(2\pi A)$.
    - In this lab, **passband gain** is ∼3 (i.e., 100/33) and the input signal has a 0.5 V amplitude (i.e., 1 V_{pp}), and so

$$f < \left(500\,\frac{\text{kV}}{\text{s}}\right)\frac{1}{2\pi(0.5\,\text{V})} \approx 159\,\text{kHz}.$$

    - Any frequencies greater than 159 kHz may look more **triangular** than sinusoidal.
      - When a device output swings at its maximum rate, it is said to be **slewing.**
      - Your *Lissajous figures* (i.e., XY-mode) may look strange in this region.
      - Do your best and use these OA limitations as part of hypotheses in your report about differences from theory.
- In second table (i.e., for *high-pass filter*), change **Calculated phase shift** from

$$-90 - \arcsin(\Delta y_{\text{intersection}}/V_{\text{o,pp}}) \qquad \text{to} \qquad \boxed{-180^\circ + \arcsin(\Delta y_{\text{intersection}}/V_{\text{o,pp}})}.$$

- Resistor color codes: Black, Brown, ROYGBV, Gray, White correspond to **digits** 0–9
  - Orange(3)-Orange(3)-Orange(3) = 33000 = 33 kΩ; Brown(1)-Black(0)-Yellow(4) = 100000 = 100 kΩ
- Capacitor codes are like resistor codes ($\text{digit}_1/\text{digit}_2$/number-of-zeros) but *typically* use unit pF
  - *Sometimes* 200 pF = `201`, but when < 1 nF, *sometimes* capacitance is codeless.
    - In today's lab 200 pF is printed as `200` (i.e., code `200` ≠ 20 pF).
    - Letters like K or M following the small capacitance are *tolerance codes* (i.e., not *units*).
- **In your report,** for **both filters**, make **semilog** plots of
  (i) **Frequency** (Hz) versus ***deciBel* gain magnitude** (i.e., $20\log_{10}(V_{\text{o,pp}}/V_{\text{i,pp}})$)
    - Make **data points** clear with plot **markers** (e.g., use `semilog(x,y,'.-')`).
    - Compare to expected magnitude curve, which is

$$|H_{\text{LPF}}(j2\pi f)| = \frac{R_2/R_1}{\sqrt{(2\pi R_2 C f)^2 + 1}}$$

or

$$|H_{\text{HPF}}(j2\pi f)| = \frac{2\pi R_2 C f}{\sqrt{(2\pi R_1 C f)^2 + 1}}.$$

    - Consider plotting expected curve on top of data curve for easy comparison.
      - In MATLAB, plot two curves using `semilog(x1,y1,'.-',x2,y2,'-')`.
      - Alternatively, try using MATLAB's `hold on` command.
  (ii) **Frequency** (Hz) versus "**calculated phase shift**" (i.e., from data table)
    - Again, make data points clear.
    - Compare to expected angle curve, which is

$$\angle H_{\text{LPF}}(j2\pi f) = -180^\circ - \arctan(2\pi R_2 C f)$$

or

$$\angle H_{\text{HPF}}(j2\pi f) = -90^\circ - \arctan(2\pi R_1 C f).$$

Be consistent about use of degrees and radians — don't mix them! (recall: $180^\circ = \pi$)
    - Again, consider overlaying expected and measured results.

For each filter, the pair of graphs represents a **Bode plot** of the system. **Try** to present possible explanations for deviations from theory (e.g., hypothesize about the impact of OA bandwidth).

Sample Code

```
%%%%%%%%%%%% Data from Measurements

% Store the frequency, measured input Vipp, output Vopp, and Lissajous vertical
% intersection data (note: the sample data here will NOT match theory below).
f = [1 2 10 20 30 40 50 60 70 80 90 100]*100;
vipp = [2.0 2.0 2.0 2.0 2.0 2.0 2.0 2.0 2.0 2.0 2.0 2.0];
vopp = [1.996 1.984 1.694 1.245 0.937 0.739 0.607 0.513 0.443 0.390 0.349 0.314];
deltaY = [0.125 0.247 0.901 0.975 0.828 0.687 0.578 0.496 0.432 0.383 0.343 0.310];

% Calculate gain vector (in dB) and phase (./ is element-by-element division).
gaindB = 20*log10( vopp./vipp );
phase = -180 - asin(deltaY./vopp)*180/pi;    % For LPF
% phase = -180 + asin(deltaY./vopp)*180/pi; % For HPF

%%%%%%%%%%%% Theoretical Predictions (from transfer function)

% For theoretical curves, 1000 j-omega points in frequency range.
ftheory = linspace( min(f), max(f), 1000 );
s = j*2*pi*ftheory;

% For transfer function, store R and C values (change as necessary!).
R1 = 33000; R2 = 100000; C = 200e-12;

% Evaluate LPF transfer function at each ftheory.
H = -(R2/R1)./(s*R2*C + 1);             % For LPF
% H = -R2*C*s./(s*R1*C + 1);            % For HPF

% Find theoretical gain (dB) and phase (degrees).
gaintheorydB = 20*log10( abs(H) );
phasetheory = angle(H)*180/pi - 360;    % For LPF (delay semantics)
% phasetheory = angle(H)*180/pi;        % For HPF

%%%%%%%%%%%% Bode plot of measurements and expectations

%%%%% Magnitude subplot: Measured and theoretical overlayed

% Put magnitude plot in top row of 2 row by 1 column figure.
subplot(2,1,1);
semilogx( f, gaindB, '.-', ftheory, gaintheorydB, '--' );   % dB Gain
grid on;                                                     % Add a grid

% Add axis labels (with units!) and title.
xlabel('Frequency (Hz)'); ylabel('Gain (dB)'); title('Gain magnitude');

%%%%% Phase subplot: Measured and theoretical overlayed

% Put phase plot in bottom row of 2 row by 1 column figure.
subplot(2,1,2);
semilogx( f, phase, '.-', ftheory, phasetheory, '--' );
grid on;

% Add axis labels (with units!) and title.
xlabel('Frequency (Hz)'); ylabel('Phase shift (degrees)'); title('Phase shift');

% Use the figure's "File" menu to save the figure in a desirable
% file format (e.g., EPS or PNG) for inclusion in your report.
```