ECE 209: Circuits and Electronics Laboratory

Notes for Lab 4 (Frequency Response of First-Order Active Circuits)

1. Comments on returned lab report.
   - Put units on tables and figures and show individual data points (i.e., not just interpolation).
   - In meters lab, positive error on R_e is a good thing. Manufacturer gives worst-case specification.

2. First-order active filters.
   - Active filters allow for gain, simplicity, robustness, and can have hard “knees” without inductors.
   - Use standard inverting/non-inverting OA configuration, but use (equivalent) Laplace-domain impedances instead of simple resistances.
   - To quickly determine characteristic of filter (i.e., low-pass, high-pass, or bandpass) consider what happens to OA configuration’s “gain” at some sample frequencies.
     - In today’s lab, both filter’s have transfer function given by \(-Z_F(s)/Z_I(s)\).
     - In the low-pass filter, \(Z_I(s) = R_1\) and \(Z_F(s) = R_2\parallel(sC)^{-1}\).
       * \(Z_F(0) \approx R_2\) for low frequencies — LF gain is \(-R_2/R_1\) (i.e., \(R_2/R_1\) with \(-180^\circ\) shift).
       * \(Z_F(j\omega) \approx 0\) for high frequencies — HF gain is \(-0/R_1 \approx 0\) (with \(-180^-90^\circ\) shift).
       * First-order filter’s time constant \(\tau\) must depend on \(C\), but because \(V_-\) is a virtual ground, the output does not “feel” the effect of \(R_1\). So time constant \(\tau = R_2C\).
     - It is a low-pass filter with passband gain \(K = -R_2/R_1\) and time constant \(\tau = R_2C\):
       \[
       H_{\text{LPF}}(s) \triangleq \frac{K}{\tau s + 1} = \frac{K\tau s}{sR_2C + 1}.
       \]
     - In the high-pass filter, \(Z_I(s) = R_1 + 1/(sC)\) and \(Z_F(s) = R_2\).
       * \(Z_I(0) \approx \infty\) (i.e., “open” for VLF) — LF gain is \(-R_2/\infty \approx 0\) (with \(-180^-90^\circ\) shift).
       * \(Z_I(j\omega) \approx R_1\) for high frequencies — HF gain is \(-R_2/R_1\) (i.e., \(R_2/R_1\) with \(-180^\circ\) shift).
       * First-order filter’s time constant \(\tau\) must depend on \(C\), but because \(V_-\) is a virtual ground, the input does not “feel” the effect of \(R_2\). So time constant \(\tau = R_1C\).
     - It is a high-pass filter with passband gain \(K = -R_2/R_1\) and time constant \(\tau = R_1C\):
       \[
       H_{\text{HPF}}(s) \triangleq \frac{Ks}{s + \frac{1}{C}} = \frac{K\tau s}{\tau s + 1} = \frac{-sR_2C}{sR_1C + 1}.
       \]
   - As in passive filters, corner frequency is where output power is half of passband power.
     - If power gain is \(1/2\), then signal gain is \(1/\sqrt{2}\). Note that \(20 \log_{10} \sqrt{2} = 10 \log_{10} 2 \approx 3\) dB.

3. Introduce and complete the Frequency Response of First-Order Active Circuits lab.
   - If 741-type operational amplifier is not available, use 747 (two 741-type OAs on one chip).
     - 747 (and 741) part pinout on supplementary document.
     - Note the supply rails! Op. amps need power from both sides to be able to work.
     - Be sure to use dual ±5 V supplies with a common ground. Use ±6 V if you see clipping.
     - If you see a lot of noise on OA output, then add 1 \(\mu\)F (code 105) across each supply rail.
       * Connect one capacitor from positive supply to ground.
       * Connect one capacitor from negative supply to ground.
   - Place capacitors close to operational amplifier pins and try to use short capacitor leads.
   - When used in this configuration, these capacitors are called bypass capacitors because they provide an alternate path for high-frequency (i.e., noisy) current. The operational amplifier sees steady rails because the ripples are directed elsewhere.
• The 741 is a SLOW OA. Its bandwidth and slew rate may corrupt high-pass filter data.
  – The bandwidth is typically 1.5 MHz, but it can be as low as 437 kHz. So signals approaching this frequency may see attenuation from poles within the OA that were previously negligible become significant. So the high-pass filter may look like bandpass filter.
  – The slew rate (i.e., maximum “speed” of OA output) is \( \sim 0.5 \text{ V/} \mu\text{s} \) (i.e., 500 kV/s).
    * “Speed” (i.e., derivative) of \( A \sin(2\pi ft) \) output is \( 2\pi Af \cos(2\pi ft) \). Maximum is \( 2\pi Af \).
    * For the OA to be able to move quickly enough, \( 2\pi Af < 500 \text{ kV/s} \); so \( f < (500 \text{ kV/s})/(2\pi A) \).
    * In this lab, passband gain is \( \sim 3 \) (i.e., 100/33) and the input signal has a 0.5 V amplitude (i.e., 1 V\(_{pp}\)), and so
      \[
      f < \left( \frac{500 \text{ kV}}{s} \right) \frac{1}{2\pi(0.5 \text{ V})} \approx 159 \text{ kHz}.
      \]
  * Any frequencies greater than 159 kHz may look more triangular than sinusoidal.
    • When a device output swings at its maximum rate, it is said to be slewing.
    • Your Lissajous figures (i.e., XY-mode) may look strange in this region.
    • Do your best and use these OA limitations as part of hypotheses in your report about differences from theory.

• In second table (i.e., for high-pass filter), change Calculated phase shift from
  \[-90^\circ - \arcsin(\Delta y_{\text{intersection}}/V_{\text{o,pp}}) \quad \text{to} \quad -180^\circ + \arcsin(\Delta y_{\text{intersection}}/V_{\text{o,pp}}).\]

• Resistor color codes: Black, Brown, ROYGBV, Gray, White correspond to digits 0–9
  – Orange-Orange-Orange = 33000 = 33 kΩ; Brown-Black-Yellow = 100000 = 100 kΩ
• Capacitor codes are like resistor codes (digit_1/digit_2/number-of-zeros) but typically use unit pF
  – Sometimes 200 pF = 201, but when < 1 nF, sometimes capacitance is codeless.
    * In today’s lab 200 pF is printed as 200 (i.e., code 200 \( \neq 20 \) pF).
    * Letters like K or M following the small capacitance are tolerance codes (i.e., not units).

• In your report, for both filters, make semilog plots of
  (i) Frequency (Hz) versus decibel gain magnitude (i.e., \( 20 \log_{10}(V_{\text{o,pp}}/V_{\text{i,pp}}) \))
    • Make data points clear with plot markers (e.g., use \( \text{semilog}(x,y',.'') \)).
    • Compare to expected magnitude curve, which is
      \[
      |H_{\text{LPF}}(j2\pi f)| = \frac{R_2/R_1}{\sqrt{(2\pi R_2 C f)^2 + 1}}
      \]
      or
      \[
      |H_{\text{HPF}}(j2\pi f)| = \frac{2\pi R_2 C f}{\sqrt{(2\pi R_1 C f)^2 + 1}}
      \]
    – Consider plotting expected curve on top of data curve for easy comparison.
    * In MATLAB, plot two curves using \( \text{semilog}(x1,y1',.'',x2,y2',.'') \).
    * Alternatively, try using MATLAB’s \texttt{hold on} command.
  (ii) Frequency (Hz) versus “calculated phase shift” (i.e., from data table)
    • Again, make data points clear.
    • Compare to expected angle curve, which is
      \[
      \angle H_{\text{LPF}}(j2\pi f) = -180^\circ - \arctan(2\pi R_2 C f)
      \]
    • Alternatively, \( \angle H_{\text{HPF}}(j2\pi f) = -90^\circ - \arctan(2\pi R_1 C f) \).
    – Be consistent about use of degrees and radians — don’t mix them! (recall: \( 180^\circ = \pi \))
    – Again, consider overlaying expected and measured results.

For each filter, the pair of graphs represents a Bode plot of the system. Try to present possible explanations for deviations from theory (e.g., hypothesize about the impact of OA bandwidth).
Sample Code

%%%%%%%%%%%% Data from Measurements

% Store the frequency, measured input Vipp, output Vopp, and Lissajous vertical
% intersection data (note: the sample data here will NOT match theory below).
f = [1 2 10 20 30 40 50 60 70 80 90 100] * 100;
vipp = [2.0 2.0 1.984 1.694 1.245 0.937 0.739 0.607 0.513 0.443 0.390 0.349];
vopp = [1.996 1.984 1.694 1.245 0.937 0.739 0.607 0.513 0.443 0.390 0.349 0.314];
deltaY = [0.125 0.247 0.901 0.975 0.828 0.687 0.578 0.496 0.432 0.383 0.343 0.310];

% Calculate gain vector (in dB) and phase (./ is element-by-element division).
gaindB = 20 * log10(vopp ./ vipp);
phase = -180 - asin(deltaY ./ vopp) * 180 / pi; % For LPF
% phase = -180 + asin(deltaY ./ vopp) * 180 / pi; % For HPF

%%%%%%%%%%%% Theoretical Predictions (from transfer function)

% For theoretical curves, 1000 j-omega points in frequency range.
ftheory = linspace(min(f), max(f), 1000);
s = j * 2 * pi * ftheory;

% For transfer function, store R and C values (change as necessary!).
R1 = 33000; R2 = 100000; C = 200e-12;

% Evaluate LPF transfer function at each ftheory.
H = -(R2 / R1) ./ (s * R2 * C + 1); % For LPF
% H = -R2 * C * s ./ (s * R1 * C + 1); % For HPF

% Find theoretical gain (dB) and phase (degrees).
gaintheorydB = 20 * log10(abs(H));
phasetheory = angle(H) * 180 / pi - 360; % For LPF (delay semantics)
% phasetheory = angle(H) * 180 / pi; % For HPF

%%%%%%%%%%%% Bode plot of measurements and expectations

%%%%% Magnitude subplot: Measured and theoretical overlayed
subplot(2,1,1);
semilogx(f, gaindB, '.-', ftheory, gaintheorydB, '--'); % dB Gain
grid on;

% Add axis labels (with units!) and title.
xlabel('Frequency (Hz)'); ylabel('Gain (dB)'); title('Gain magnitude');

%%%%% Phase subplot: Measured and theoretical overlayed

% Put phase plot in bottom row of 2 row by 1 column figure.
subplot(2,1,2);
semilogx(f, phase, '.-', ftheory, phasetheory, '-.');
grid on;

% Add axis labels (with units!) and title.
xlabel('Frequency (Hz)'); ylabel('Phase shift (degrees)'); title('Phase shift');

% Use the figure's "File" menu to save the figure in a desirable
% file format (e.g., EPS or PNG) for inclusion in your report.